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The Curse of Low-valued Recycling

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Abstract

This paper discusses how to deal with low-valued recyclable residual wastes whose reprocessing itself does not pay financially. While such a recycling activity can potentially improve the social welfare if the environmental costs associated with its disposal are sufficiently significant, governmental policies to promote recycling may be taken advantage of and lead to even more harmful consequences, such as illegal dumping. By constructing a model that includes both disposal and recycling activities and, furthermore, by explicitly considering the government's monitoring cost in preventing firms from disposing of collected wastes illicitly, we identify the second-best deposit-refund (D-R) policy for a low-valued recyclable. Our results indicate, however, that in implementing such a policy a policy-maker has to face critical informational issues, which is in stark contrast to the D-R policy for a non-low-valued recyclable.

Keywords: deposit-refund; illegal waste disposal; monitoring; recycling

JEL Classification: H21; Q21; Q28

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1 Introduction

In the presence of a household's incentive to illegally dispose of its waste, a so-called "two-part instrument (2PI)" or "deposit-refund (D-R) scheme" is considered to be a more effective policy tool over others.[1] To the best of our knowledge, however, most of the previous studies have not paid close attention to the possibility that commercially-transacted recyclable residual wastes do not actually get recycled and, instead, illegally disposed by firms.[2] Illegal disposal can be a profitable option to firms when the government provides a subsidy for simply obtaining wastes in a recycling market.

Ino (2011) examines the implications of such illicit behaviors by firms in determining the optimal policy levels, and finds that the two-part instrument is still reasonably effective as long as the net private benefit of recycling is positive, at least, up to a certain extent. However, this condition does not always hold in reality. While the recycling of containers and packaging have recently seen increasing governmental involvement in the forms of taxes and subsidies in some developed countries, recycled materials from used PET bottles and glass containers currently have very small economic values, as opposed to, say, aluminum cans. These low-valued recyclables may get recycled solely due to the presence of these policies, and the mere fact of private firms' participating in residual waste trades does not imply that the recycling is socially desirable. Indeed, by carefully estimating the net social benefit of recycling household solid wastes, Kinnaman et al. (2014) report that the welfare-maximizing recycling rates would be well below the observed and mandated recycling rates for Japan and perhaps for other developed nations as well.

Moreover, when the recycled materials have very low economic values, this potential "over-encouragement" of recycling can easily lead to firms's illegal disposal of wastes that are initially intended for recycling. In Japan, for instance, numerous midnight dumping cases of cleansed and properly-sorted PET bottles in large chunks have been reported in the

[1] Fullerton and Kinnaman (1995) and Palmer and Walls (1997) are seminal works in this literature. Fullerton and Wolverton (2005) show its effectiveness in a more general context.

[2] On the other hand, illegal disposal by households has been carefully examined by Fullerton and Kinnaman (1995) and Choe and Frazer (1999), for instance.

news media since the implementation of the Law for the Promotion of Sorted Collection and Recycling of Containers and Packaging (or, Container and Packaging Recycling Law, in short) in 1997. Given the typical volumes of those wastes, it is obvious that they were discarded by firms and not by households. The law is intended to make the manufacturers of packaging and the retailers of packaged products financially responsible for the disposal of the packaging wastes and also to encourage the recycling of those wastes by providing extra monetary incentives for recyclers. It is considered, however, that some recyclers have taken advantage of the law and received illegitimate financial benefits for the recycling activities they have not conducted properly.

On the other hand, it is not necessarily the case that such "low-valued recycling" is inefficient from a society's perspective, especially when the disposal cost of the waste is large. Recycling can be justified on the social welfare ground even if a recycled material *per se* has a fairly low market value. In this paper, we attempt to derive the optimal policy set under the condition that the recycled material has such a small market value that makes the net private benefit of recycling negative for a recycler. Then, based on the structure of this optimal policy, we discuss why it is considerably more difficult to actually implement the second-best deposit-refund policy for a low-valued recyclable product.

The paper is organized as follows. The next section describes our economic model, and Section 3 derives the second-best policy set. The following section discusses the reason why, in the case of low-valued recyclables, it is difficult, especially on the informational grounds, for a policy-maker to implement the second-best policy without over-encouraging recycling activities which results in the creation of a socially inefficient recycling market and the illegal waste disposal.

2 The Model

We essentially adopt a modified version of the partial equilibrium model in Ino (2011), which includes both the product and recyclable residual waste markets. The proper completion of recycling requires the reprocessing efforts by households, such as cleansing, sorting-out and storing the wastes, as well as by firms. For simplicity, we consider only one representative household, and one representative firm which plays the roles of both a recycler and a producer.[3] The household and the firm are supposed to be price-takers.

We assume that one unit of the product generates one unit of recyclable waste after consumption. The authorities provide the legal waste collection service for the household, and the unit charge for this service equals $\tau_h \in \mathbb{R}_+$. Potentially, the household can dispose of its residual wastes illegally to avoid this unit charge. In order to mitigate such an incentive, the authorities can monitor illegal disposal activities by the household, besides providing the legal waste collection service. With sufficiently high levels of monitoring and fines, the household will not resort to illicit disposal options even when there is some positive unit charge on legal disposal. If the marginal social cost of the illegally disposed waste is greater than that of legally disposed one as we assume in this paper[4], such a level of monitoring is optimal from the viewpoint of social welfare maximization. Indeed, we can show that under our optimal policy set all the waste disposal by the household ends up being conducted legally.[5] Then, the following simple material balance condition applies: $z = x^d - r^s$, where z is the amount of waste legally disposed by the household, x^d the household's demand for the product, and r^s the household's supply of the recyclable waste to the firm. We consider the case where $x^d > 0$ and $r^s \geq 0$ (hence, the boundary case of zero recycling by the household is allowed). Moreover, to facilitate the analysis, we assume $r^s < x^d$; that is, we exclude the case where z is zero.[6]

With the use of a quasi-linear utility function, we assume that the representative household's behavior is approximated by the following constrained utility maximization problem with respect to x^d, r^s, and a numeraire, y:

$$\max_{x^d, r^s, y} U(x^d) + y, \tag{1}$$

[3]All the results and implications of this paper can be extended to an economy with $m \geq 1$ firms and $n \geq 1$ households and also to the case where recyclers and output producers are separate entities.

[4]This assumption is also adopted by Fullerton and Kinnaman (1995) and Choe and Frazer (1999).

[5]For the formal analysis including the proof of this fact, see Ino (2011).

[6]This assumption let us focus on a realistic case where, with currently available recycling technologies, it is not socially optimal to attain zero waste disposal.

$$\text{s.t.} \;\; P_x x^d + y + \tau_h(x^d - r^s) + C_r(r^s) \leq I + P_r r^s, \tag{2}$$

where I is the household's income, $P_x \in \mathbb{R}_{++}$ the price of the product, and $P_r \in \mathbb{R}$ the price of the recyclable waste.[7] Also, $U : \mathbb{R}_+ \mapsto \mathbb{R}$ signifies the sub-utility function of the household, which is strictly increasing and strictly concave, and $C_r(r^s) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ is a strictly increasing and strictly convex cost function associated with the household's separation and other activities that are necessary for proper recycling by the firm later.[8]

Then, the first-order conditions give us the household's inverse demand function for the product and its inverse supply function of the recyclable residual wastes, respectively:

$$P_x(x^d; \tau_h) = U'(x^d) - \tau_h, \tag{3}$$

$$P_r(r^s; \tau_h) = C'_r(r^s) - \tau_h, \tag{4}$$

for all $x^d > 0$ and $r^s \geq 0$. Here, we define $P_r(0; \tau_h) \equiv \lim_{r^s \to 0} P_r(r^s; \tau_h) = C'_r(0) - \tau_h$.

The representative firm produces a recyclable product with some technology that is represented by a strictly increasing and strictly convex cost function, $C_x(x^s) : \mathbb{R}_+ \mapsto \mathbb{R}_+$, where x^s is the firm's supply of the product. The firm also demands the recyclable household wastes by the amount of r^d. After the firm obtains the residual wastes, it might illegally dispose of the wastes, instead of recycling them properly. We denote the amount of the firm's illegal waste disposal by z^f, and r^c is the quantity of residual wastes that are completely reprocessed by the firm. Again, we suppose a simple material balance condition for the firm as well: $z^f = r^d - r^c$.

The firm's net benefit of proper recycling is given by a strictly concave function, $B(r^c) : \mathbb{R}_+ \mapsto \mathbb{R}$. Note that we allow the value of $B(r^c)$ to be negative since it contains the cost of reprocessing the residual wastes.[9] Indeed, in order to focus on the issues arising when

[7] When $P_r > 0$ ($P_r < 0$), the firm (household) pays in the recyclable residual market. Note that P_r can be negative because the household may still be willing to pay in order to avoid the charge for waste disposal.

[8] Kinnaman et al. (2014) empirically estimate the recycling cost to households.

[9] In our model, $B(r^c)$ is given exogenously, and it can be interpreted in several different ways. Here, $B(r^c)$ is defined as $B'(r^c) = p - C'(r^c)$, where p is the exogenously given price of the recycled material and $C'(r^c)$ is the reprocessing cost. If the firm sells its recycled material in some other market, p is simply the market price of the material. If the firm uses the recycled material as an input for its own production, p is the price of a perfectly substitutable virgin input, v, provided that the production function of the good x is of the form $x = f(r + v)$.

the recycled material has fairly low economic value, we assume below that $B'(0) \leq 0$.[10] Furthermore, we suppose that $\lim_{r\to\infty} B'(r) = -\infty$.

By choosing the stringency of their monitoring activities, the authorities essentially control the expected unit penalty on the firm's illegal disposal, $\tau_f \in \mathbb{R}_+$. The level of the expected penalty is determined by multiplying the unit penalty by the probabilities of detecting illegal disposal activities. Raising the probability of detection requires more patrol efforts, which leads to an increase in the monitoring cost. We suppose that it costs the authorities $\Gamma_f(\tau_f) : \mathbb{R}_+ \mapsto \mathbb{R}_+$ to monitor the firm's disposal activities. On the other hand, their cost of monitoring the household's illegal waste disposal is given by $\Gamma_h(\tau_h) : \mathbb{R}_+ \mapsto \mathbb{R}_+$. This cost is a function of a unit charge on its legal disposal because a higher disposal charge requires a higher level of monitoring on the household so as to achieve zero illegal disposal there.[11] Both Γ_h and Γ_f are strictly increasing and convex functions.

As additional policy tools, we consider two policy instruments which could be called a "deposit-refund (D-R)" scheme in combination: the tax $t \in \mathbb{R}$ (deposit) in the product market and the subsidy $s \in \mathbb{R}$ (refund) in the residual waste market. The total tax and subsidy amounts are proportional to the quantities of the market-exchanged product and its residual wastes, x^s and r^d, respectively. As an important assumption, we suppose that the subsidy on the commercially-transacted residual wastes is provided without knowing exactly how much of them the firm will eventually reprocess into recycled materials. Although the market-based information on the firm (that is, x^s in the product market, and r^d in the recyclable residual waste market) is available and verifiable for the policy maker, the amount of residual wastes actually reprocessed by the firm, i.e., r^c, cannot be observed because this constitutes the firm's private information.[12] Therefore, the firm might surrep-

[10]The case where $B'(0) > 0$ is analyzed in Ino (2011).

[11]As is mentioned above, zero illegal disposal by the household is always optimal from the society's viewpoint.

[12]Thus, we suppose that the total amount of the transacted residual wastes, r^d, which is observable in the recycling market, is separated into two unobservable variables, illegally disposed wastes, z^f, and properly reprocessed ones, r^c, according to the material balance equation, $r^d = z^f + r^c$. In the terms typically found in the environmental economics literature, r^d and r^c respectively correspond to the reported and actual levels of emission abatement while z^f is the level of cheating by the firm. While in this paper we adopt the notion that the authorities attempt to detect the cheating by directly monitoring the firm's illicit activity, z^f can also be identified indirectly by observing its legal activity, r^c, in the light of the material balance equation.

titiously get rid of its obtained residual wastes and pretend to have reprocessed the wastes properly.

Finally, the waste disposal activities of both the household and the firm, i.e., z and z^f, respectively generate constant marginal social costs of $d \in \mathbb{R}_{++}$ and $d_f \in \mathbb{R}_{++}$, including the cost associated with certain environmental damages. Since illegal disposal, such as midnight dumping and illicit burning, would typically be socially more costly than legal disposal options, such as controlled landfills and proper incineration, we assume $d < d_f$. Then, the social welfare function W is defined as

$$W \equiv [U(x^d) - P_x x^d] + [P_r r^s - C_r(r^s)] - \tau_h z \tag{5}$$

$$+ [(P_x - t)x^s - C_x(x^s)] + [B(r^c) - (P_r - s)r^d] - \tau_f z^f \tag{6}$$

$$+ \tau_h z + \tau_f z^f + t x^s - s r^d - \Gamma_h(\tau_h) - \Gamma_f(\tau_f) - dz - d_f z^f. \tag{7}$$

Here, (5) and (6) are the consumer surpluses and the producer surpluses, respectively. Note that the second brackets of (5) and (6) are the surpluses related to the reprocessing/recycling activities, and the third terms are the payments associated with waste disposal. Finally, (7) signifies the sum of the authorities' tax and fine revenues, subsidy and monitoring costs, and the social costs associated with waste disposal. With the two market clearing conditions, i.e., $x^d = x^s$ and $r^d = r^s$, we can rewrite the social welfare W simply as

$$W = [U(x^d) - C_r(r^s) - dz] + [B(r^c) - C_x(x^s) - d_f z^f] - \Gamma_h(\tau_h) - \Gamma_f(\tau_f). \tag{8}$$

Indeed, our model can also be used to depict the latter case, where Γ_f is now interpreted as the monitoring cost on the firm's proper recycling efforts. Even in such a case, we argue that, typically, it is not costless to trace the whole recycling processes for the following reasons. First, the residual wastes of one product can be reprocessed into a variety of recycled materials by different firms, which implies that it would be difficult to verify the actual usage of recycled materials. Second, the recycled materials are often mixed with some virgin materials in the production process, which suggests that it is not costless to identify the actual ratio of recycled materials used. Third, the firm may internally use the recycled materials in its own production, which implies that the market-based information on the exact usage of recycled materials may not be available. If it is extremely easy to trace the whole recycling processes in spite of these facts, that is, if Γ_f is infinitesimally small, our model becomes close to the first-best situation (see Footnote 18).

3 The Optimal Policy Set

We solve the optimal policy choice problem in steps. First, given the policy variables, $(t, s; \tau_f, \tau_h)$, we derive the market equilibrium. Then, we find the monitoring level that achieves zero illegal disposal by the firm. In particular, we call such a monitoring level as the "optimal monitoring rule." As we will see below in Lemma 1, this is admissible in deriving the optimal policy set due to the assumption that the illegal waste disposal by the firm is always socially more costly than the legal waste disposal by the household. Thus, we can focus our attention on the equilibrium quantities and prices that are induced under the optimal monitoring rule. Finally, we obtain the second-best policy set that maximizes the social welfare.

3.1 Behavior of the Firm and the Market Equilibrium

The profit maximization problem of the firm is given by

$$\begin{aligned} &\max_{x^s, r^c, r^d, z^f} [(P_x - t)x^s - C_x(x^s)] + [B(r^c) - \tau_f z^f - (P_r - s)r^d] \\ &\text{s.t. } r^d = z^f + r^c. \end{aligned} \tag{9}$$

Then, the first-order conditions are

$$P_x = C_x{}'(x^s) + t, \tag{10}$$

$$B'(r^c) \leq \mu \quad \text{with equality if} \quad r^c > 0, \tag{11}$$

$$\mu \leq P_r - s \quad \text{with equality if} \quad r^d > 0, \tag{12}$$

$$-\tau_f \leq \mu \quad \text{with equality if} \quad z^f > 0, \tag{13}$$

where μ is the Lagrangian multiplier associated with the constraint. Presuming $x^s > 0$, (10) gives the inverse supply function of the product.

For the time being, let us suppose that both s and τ_f are exogenously given. From (11) – (13) and the constraint, the demand correspondence for the recyclable waste, i.e., r^d, is described as follows.

When (i) $B'(0) \geq -\tau_f$, the demand is

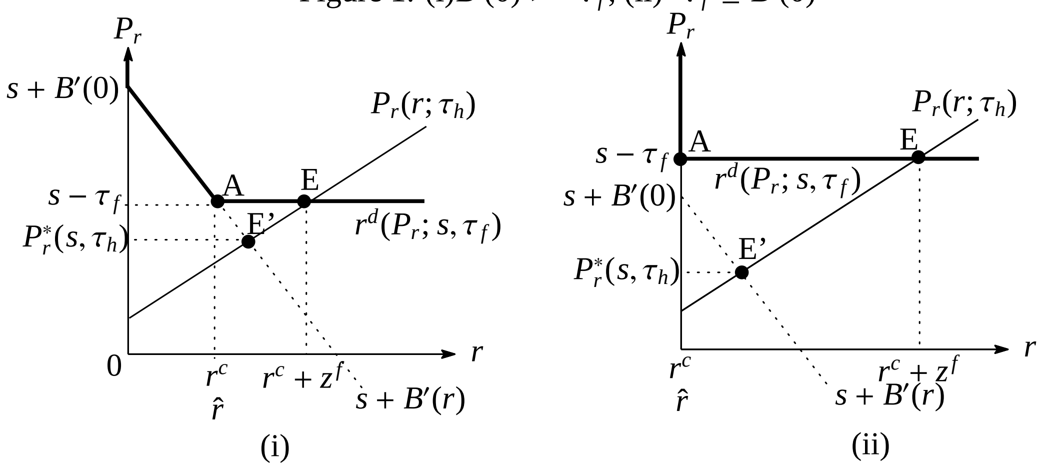
$$r^d(P_r; s, \tau_f) = \begin{cases} 0 & \text{if} \quad P_r \geq s + B'(0), \\ B'^{-1}(P_r - s) & \text{if} \quad s + B'(0) \geq P_r \geq s - \tau_f, \\ [\hat{r}(\tau_f), \infty) & \text{if} \quad P_r = s - \tau_f, \end{cases} \tag{14}$$

where $\hat{r}(\tau_f) = B'^{-1}(-\tau_f)$. Thus, $\hat{r}(\tau_f)$ is the residual wastes' amount where the net marginal benefit of proper recycling, $s + B'$, which can be derived from (11) and (12), equals the net marginal benefit of illicit disposal for the firm, $s - \tau_f$, which can be derived from (12) and (13). When (ii) $-\tau_f > B'(0)$, on the other hand, the demand correspondence is

$$r^d(P_r; s, \tau_f) = \begin{cases} 0 & \text{if} \quad P_r \geq s - \tau_f, \\ [\hat{r}(\tau_f), \infty) & \text{if} \quad P_r = s - \tau_f, \end{cases} \tag{15}$$

where $\hat{r}(\tau_f) = 0$. For the respective cases, the demand curves in the residual waste market are drawn in bold lines in Figure 1, where $P_r(r; \tau_h) = C_r' - \tau_h$ denotes the inverse supply curve given by (4).

Figure 1: (i)$B'(0) > -\tau_f$, (ii)$-\tau_f \geq B'(0)$



The equilibrium outcomes are determined by the intersections of the demand and supply curves in the product and residual waste markets. In the waste market, the equilibrium is given by the point E in Figure 1. Since $s + B'(r)$ is the net marginal benefit of proper recycling, and $s - \tau_f$ is the net marginal benefit of illegal disposal, the wastes that are obtained by the firm are completely recycled when $r^d \leq \hat{r}(\tau_f)$, whereas the firm disposes of its wastes by the amount which exceeds $\hat{r}(\tau_f)$ when $r^d > \hat{r}(\tau_f)$. In Figure 1, the recycled amount of the waste is represented by the segment to the left of the kinked point A, while the illegally disposed amount is given by the segment to the right of A until it hits the supply curve at the point E. Particularly, in case (ii), all the transacted wastes eventually end up being disposed illegally by the firm.

3.2 The Optimal Monitoring Rule

Let $z_f^*(t, s; \tau_f, \tau_h)$ be the equilibrium level of the firm's illegal disposal under a policy set $(t, s; \tau_f, \tau_h)$, then we can obtain the following result:

Lemma 1. *When* $z_f^*(t, s; \tau_f, \tau_h) > 0$ *for a given policy set, there exists an alternative policy set that improves welfare.*

Proof. See the Appendix. **Q.E.D.**

Thanks to this lemma, in deriving the optimal set of policies, we can exclude the cases where the firm illegally disposes of the residual wastes it has obtained, and focus on the monitoring level that achieves $z_f^* = 0$. In order to prevent any illegal disposal, the authorities basically need to maintain the monitoring level sufficiently high. In addition, they should select the lowest among such sufficient monitoring levels to save on the monitoring cost. Specifically, the authorities should maintain the monitoring efforts at a level that is high enough to satisfy *the optimal monitoring rule* $\hat{\tau}_f(s; \tau_h)$ defined by

$$\hat{\tau}_f(s; \tau_h) = \begin{cases} s - P_r^*(s, \tau_h) & \text{if} \quad s > P_r^*(s; \tau_h), \\ 0 & \text{if} \quad s \leq P_r^*(s; \tau_h), \end{cases} \tag{16}$$

where $P_r^*(s, \tau_h)$ is the equilibrium price of the residual wastes that is induced under the condition that illegal disposal from the firm is zero; that is, $P_r^*(s, \tau_h) \equiv P_r(r^*(s; \tau_h); \tau_h)$, where $r^*(s; \tau_h)$ is such an amount of the residual wastes that satisfies

$$P_r(r^*(s; \tau_h); \tau_h) = B'(r^*(s; \tau_h)) + s, \tag{17}$$

if $P_r(0; \tau_h) < B'(0) + s$, and $r^*(s; \tau) = 0$ if $P_r(0; \tau_h) \geq B'(0) + s$. Note that the left-hand side of (17) is inverse supply of residual wastes and the right-hand side of that is the marginal benefit of proper recycling. Thus, (17) is the market clearing condition under proper recycling and identifies the equilibrium point where there is no illegal disposal, which is shown by the point E' in Figure 1.

When $s > P_r^*$ as in the first line of (16), the subsidy for transacted residual wastes is higher than the price if the firm is properly engaged in recycling. Hence, the firm has the incentive to purchase the residual wastes further and dispose of them illegally after pocketing the subsidy. To remove this incentive, the expected penalty on illegal disposal should completely offset the subsidy-price margin, $s - P_r^*$, as is stated in the optimal monitoring rule. When $s \leq P_r^*$ as in the second line of (16), since the subsidy-price margin is negative, the firm does not have the incentive to purchase the residual wastes solely for the sake of illegal disposal and thus, the optimal monitoring level is zero.[13] As a result, when $\tau_f = \hat{\tau}_f(s; \tau_h)$, the firm's illegal disposal z_f^* is zero and thus the equilibrium amount of the recycled wastes is $r^*(s; \tau_h)$. It then follows that, under the optimal monitoring rule, the equilibrium amount of legally disposed household's wastes is $z^*(t, s; \tau_h) = x^*(t; \tau_h) - r^*(s; \tau_h)$, where $x^*(t; \tau_h)$ represents the equilibrium amount of the product that is given by (3) and (10):

$$P_x(x^*(t; \tau_h); \tau_h) = C_x{}'(x^*(t; \tau_h)) + t. \tag{18}$$

Now, we can identify the following two key threshold levels of the subsidy, $\bar{s}$ and $\hat{s}$, for the firm. The threshold $\bar{s}$ is the smallest amount of the subsidy that could induce the firm to recycle the wastes properly, provided that its incentive to dispose of the wastes illicitly is sufficiently curtailed. The threshold $\hat{s}$ is the largest subsidy level under which no monitoring cost incurs.

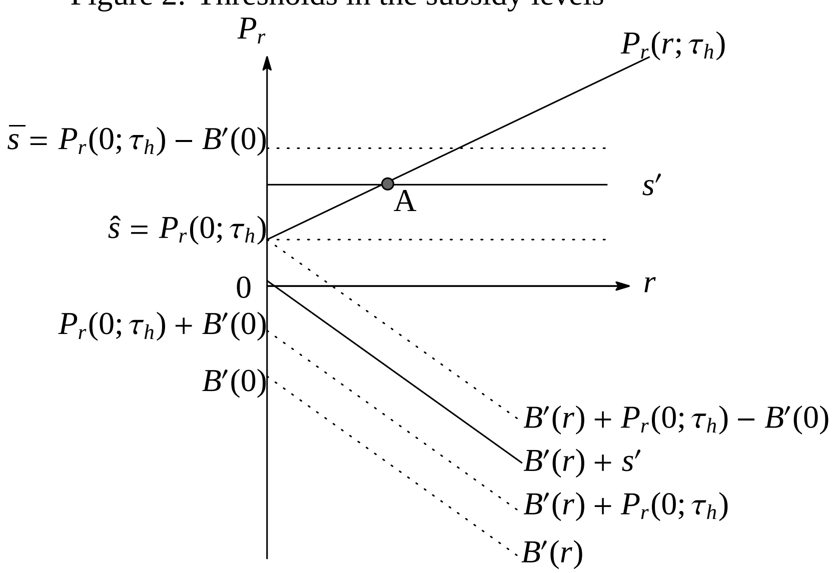
Lemma 2. *(i)* $r^*(s; \tau_h) = 0$ *if and only if* $s \leq \bar{s} \equiv P_r(0; \tau_h) - B'(0) = C_r'(0) - \tau_h - B'(0)$. *(ii)* $\hat{\tau}_f(s; \tau_h) = 0$ *if and only if* $s \leq \hat{s} \equiv P_r(0; \tau_h) = C_r'(0) - \tau_h$.

[13]Under the optimal monitoring rule, point A in Figure 1 is exactly on the supply curve of the residual wastes at $P_r^*(s; \tau_h)$ when $\hat{\tau}_f > 0$ and below the supply curve when $\hat{\tau}_f = 0$. Thus, when $\tau_f \neq \hat{\tau}_f$, the authorities can improve the welfare by eliminating the firm's illegal disposal (when $\tau_f < \hat{\tau}_f$) or by saving on the monitoring cost (when $\tau_f > \hat{\tau}_f$). It is important to note that the optimal monitoring rule works as well even if $r^* = 0$ ($P_r(0; \tau_h) \geq B'(0) + s$) since $P_r^*(s; \tau_h) = P_r(0; \tau_h) = C_r'(0) - \tau_h$.

Proof. (i) It immediately follows from the definition of $r^*(s;\tau)$ that $r^*(s;\tau_h) = 0$ if and only if $P_r(0;\tau_h) \geq B'(0) + s$, or equivalently, $s \leq P_r(0;\tau_h) - B'(0)$. (ii) First, consider the case where $s \leq P_r(0;\tau_h) - B'(0)$. In this case, we have $r^*(s;\tau_h) = 0$. This implies that $P_r^*(s;\tau_h) = P_r(0;\tau_h)$. Thus, $\hat{\tau}_f(s;\tau_h) > 0$ if and only if $s > P_r^*(s;\tau_h) = P_r(0;\tau_h)$ by the definition of (16). Next consider the case where $s > P_r(0;\tau_h) - B'(0)$. In this case, $s > P_r(0;\tau_h)$ is always obtained by $B'(0) \leq 0$. Furthermore, in this case, we have $r^*(s;\tau_h) > 0$. Hence, $P_r^*(s;\tau_h) = B'(r^*(s;\tau_h)) + s$. Since $B'(r^*(s;\tau_h)) < 0$, it follows that $P_r^*(s;\tau_h) < s$ and thus $\hat{\tau}_f(s;\tau_h) > 0$ by the definition of (16). **Q.E.D.**

Because we suppose $B'(0) \leq 0$, $\hat{s} \leq \bar{s}$ always holds (with strict inequality if $B'(0) < 0$). Therefore, when $\hat{s} < s \leq \bar{s}$, as is seen in the case where $s = s'$ in Figure 2, both $r^*(s;\tau_h) = 0$ and $\hat{\tau}_f(s;\tau_h) > 0$ are satisfied. In such a situation,[14] even though the residual waste market does not exist, the authority must conduct some monitoring on the firm because, without monitoring ($\tau_f = 0$), the waste market emerges and all the wastes obtained by the firm are illegally discarded (i.e., all the traded residual wastes of the amount A are disposed of illegally).

Figure 2: Thresholds in the subsidy levels



[14]Observe that in the figure, the marginal benefit of illegal disposal s' intersects the supply curve of the residual wastes $P_r(r,\tau_h)$, but the marginal benefit of proper recycling $B'(r) + s'$ does not.

3.3 The Optimal Policy Set

Given the optimal monitoring rule and the resulting equilibrium outcomes, we now proceed to the final step, where we determine the optimal policy set $(t^*, s^*; \tau_h^*, \tau_f^*)$ so as to maximize the social welfare under the condition $\tau_f^* = \hat{\tau}_f(s^*; \tau_h^*)$. The welfare maximization problem is formally described as:

$$\max_{t,s;\tau_h} W^*(t, s; \tau_h) \equiv [U(x^*(t; \tau_h)) - C_r(r^*(s; \tau_h))] + [B(r^*(s; \tau_h)) - C_x(x^*(t; \tau_h))] - d[x^*(t; \tau_h) - r^*(s; \tau_h)] - \Gamma_h(\tau_h) - \Gamma_f(\hat{\tau}_f(s; \tau_h)). \quad (19)$$

The solution to this problem gives us the second-best policy set.[15] First, we show that the authorities should not charge the household for the waste disposal service.

Proposition 1. *Under the optimal policy set, the authorities always set* $\tau_h^* = 0$.

Proof. See the Appendix. **Q.E.D.**

For obtaining the optimal policy set, therefore, we can focus on the case where $\tau_h = 0$. Henceforth, we omit τ_h from the arguments unless it is necessary.

Second, we show that the optimal product tax (or, the deposit on a unit of the product) is equal to the marginal social cost of the legally disposed wastes by the household.

Proposition 2. *Under the optimal policy set, the authorities always set* $t^* = d$.

Proof. See the Appendix. **Q.E.D.**

The next step is to derive the optimal subsidy level with $\tau_h = 0$ and $t = d$. To solve the problem (19) with respect to s, we divide the situation into the following two cases depending on whether the residual waste market emerges or not (note that $P_r(0; 0) = C_r'(0)$).

[15] We assume that the appropriate second-order conditions with respect to (t, s) are globally met when $x^* > 0$ and $r^* > 0$.

Proposition 3. *If $r^*(s^*; 0) > 0$ under the optimal policy set, the subsidy level is given by $s^* = s^M$, which satisfies $s^M = d - A(s^M)$, where*[16]

$$A(s^M) \equiv -B''(r^*(s^M; 0))\Gamma_f'(s^M - P_r^*(s^M; 0)) > 0.$$

If $r^*(s^*; 0) = 0$ under the optimal policy set, the optimal subsidy is any s that satisfies $s \leq C_r'(0)$.

Proof. See the Appendix. **Q.E.D.**

Finally, we show the condition under which the emergence of the recycling market is socially desirable. Along with the definition of the optimal monitoring rule in the previous subsection and the optimal subsidy levels described in the proposition just above, this condition gives us the threshold level of d that distinguishes between different patterns for monitoring the firm as a part of optimal policy set.

Proposition 4. *There exists $\bar{d} \geq \bar{s} + A(\bar{s})$ (with equality if and only if $B'(0) = 0$)*[17] *such that $r^*(s^*; 0) > 0$ under the optimal policy if and only if $d > \bar{d}$. Thus, the optimal expected penalty on the firm is $\tau_f^* = s^M - P_r^*(s^M; 0) > 0$ if $d > \bar{d}$; and $\tau_f^* = 0$ if $d \leq \bar{d}$.*

Proof. Choose the value of d that satisfies $s^M = \bar{s} = Cr'(0) - B'(0)$ and $d = \bar{s} + A(\bar{s})$, and denote it by d', where

$$A(\bar{s}) \equiv -B''(r^*(\bar{s}))\Gamma_f'(\bar{s} - P_r^*(\bar{s})) = -B''(0)\Gamma_f'(-B'(0)).$$

since $r^*(\bar{s}) = 0$ and $P_r^*(\bar{s}) = P_r(0) = C_r'(0)$. Consider the case where $d > d'$. Then, $s^M > \bar{s}$ holds. This is because, if $s^M \leq \bar{s}$, $d \leq d'$ must hold since

$$d = s^M + A(s^M) = s^M - B''(0)\Gamma_f'(s^M - C_r'(0)) \leq s^M - B''(0)\Gamma_f'(-B'(0)) \leq d'.$$

[16] Note that, in finding the actual level of s^M, we need to know $A(s^M)$, which includes the information that is available only in the residual waste market because we have the relation:

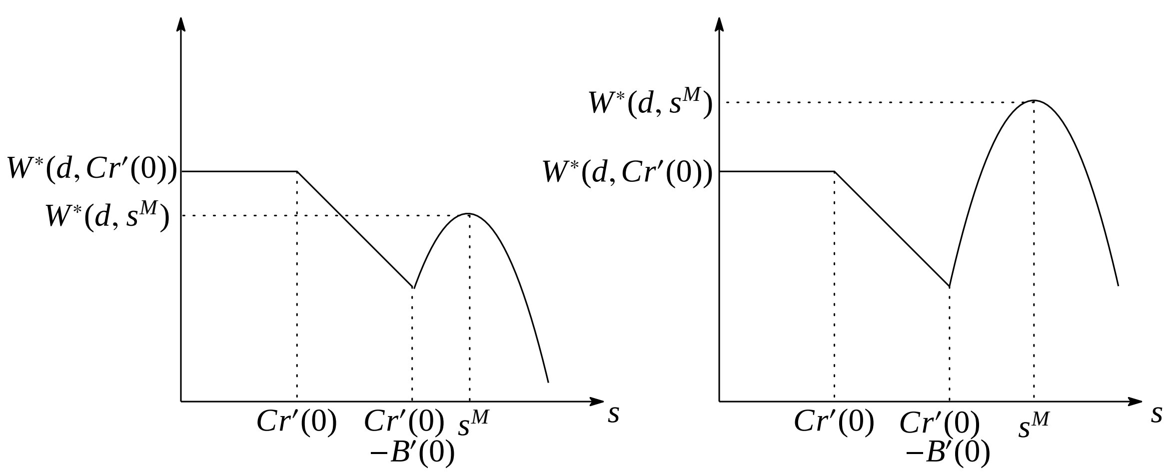
$$B''\Gamma_f' = \frac{dP_r}{dr^d}\Gamma_f' = \frac{P_r\Gamma_f'}{r}\left(\frac{dP_r/P_r}{dr^d/r}\right) = \frac{P_r\Gamma_f'}{r\eta_r^d},$$

where η_r^d represents the elasticity of the demand for the residual wastes.

[17] As is in the proof, in order to induce $\bar{d}$, we must compare two locally maximized welfare levels, $W^*(d, s^M)$ and $W^*(d, C_r'(0))$. Thus, $\bar{d}$ consists of the market information that is only available in the residual waste market.

as $s^M \leq \bar{s}$ and $\Gamma_f'' \geq 0$. In this case, we must compare two locally maximized welfare levels, $W^*(d, s^M)$ and $W^*(d, C_r'(0))$ (see Figure 3). Due to the envelope theorem and the fact that $r^*(s^M) > 0$ as $s^M > \bar{s}$, we have $\partial W^*(d, s^M)/\partial d = r^*(s^M) - x^*(d)$. Also from the envelope theorem and the fact that $r^*(C_r'(0)) = 0$ as $C_r'(0) \leq \bar{s}$, we have $\partial W^*(d, C_r'(0))/\partial d = r^*(C_r'(0)) - x^*(d) = -x^*(d)$. Thus, (i) $W^*(d, s^M) - W^*(d, C_r'(0))$ is strictly increasing in $d > d'$. Now take an arbitrary s such that $s > \bar{s}$. Then, $W^*(d, s^M) \geq W^*(d, s)$ since $s = s^M$ gives the local maximum in this range. Then, $\partial(W^*(d, s) - W^*(d, C_r'(0)))/\partial d = r^*(s) > 0$, where $r^*(s)$ is constant for any d. Hence, if d is sufficiently large, (ii) there exists $d > d'$ such that $W^*(d, s^M) - W^*(d, C_r'(0)) \geq W^*(d, s) - W^*(d, C_r'(0)) > 0$. Furthermore, (iii) $W^*(d, s^M) - W^*(d, C_r'(0)) \leq 0$ holds if $d = d'$ (with equality if and only if $B'(0) = 0$) since $W^*(d, \bar{s}) \leq W^*(d, C_r'(0))$ (note that $W^*(d, s)$ is decreasing in s when $C_r'(0) < s \leq \bar{s}$ as shown in the proof of Lemma 3). From (i)-(iii), there exists a value of $\bar{d} \geq d'$ such that $W^*(d, s^M) - W^*(d, C_r'(0)) > 0$ if and only if $d > \bar{d}$, where $\bar{d} = d'$ if and only if $B'(0) = 0$. Consider the case where $d \leq d'$. Then, $W^*(d, s)$ is decreasing in s when $s > \bar{s}$ since $s^M \leq \bar{s}$. Therefore, under the optimal policy, $s^* \leq C_r'(0)$ and $r^*(s^*) = 0$. **Q.E.D.**

Figure 3: Welfare comparison

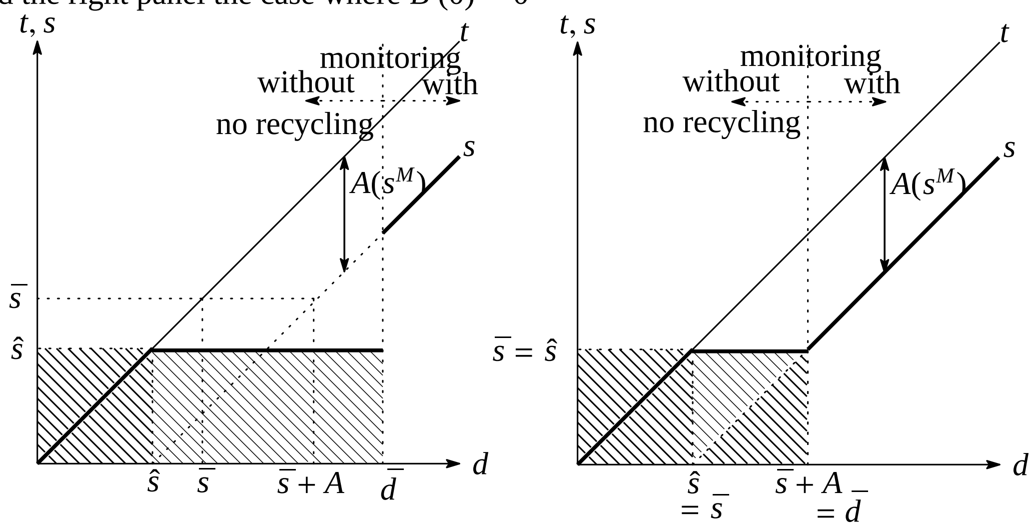


cases	$d > \bar{d}$	otherwise
tax t	$t^* = d$	$t^* = d$
subsidy s	$s^* = d - A(s^M) > C_r'(0)$	$s^* \leq C_r'(0)$
monitor τ_f	$\tau_f^* = s^M - P_r^*(s^M) > 0$	$\tau_f^* = 0$
fee τ_h	$\tau_h^* = 0$	$\tau_h^* = 0$
recycling r	$r^*(s^*; \tau_h^*) > 0$	$r^*(s^*; \tau_h^*) = 0$

Table 1: Optimal policy set and recycling

Propositions 1-4 gives us the second-best policy structure. For convenience, Table 1 summarizes the results of these propositions, and Figure 4 depicts the pattern of the optimal tax and subsidy for the case where $B'(0) < 0$ and the case where $B'(0) = 0$, respectively.

Figure 4: Pattern of the optimal policy: The left panel depicts the case where $B'(0) < 0$ and the right panel the case where $B'(0) = 0$



The optimal policy set in the case where $d > \bar{d}$ (the second column of Table 1 and the right-hand region of each panel in Figure 4 that is denoted by "monitoring-with") is the system of imposing the tax on the product (deposit) for the full social cost of the legal disposal, a somewhat reduced subsidy to the commercially transacted residual wastes (refund), and free disposal service for legally disposed household waste to avoid conducting any monitoring on the household. This is the D-R policy modified by taking into account the monitoring cost on the firm.[18] Since the environmental damage d is sufficiently large in this case, the authorities should encourage the firm to undertake recycling by providing the appropriate amount of subsidy ($s^* > \hat{s}$: the optimal levels of subsidy should be on the bold line of Figure 4) and implementing the necessary monitoring on the firm.

In the case where $d \leq \bar{d}$ (the third column of Table 1 and the left-hand region of each panel in Figure 4 that is denoted by "monitoring-without"), however, the environmental damage d is small enough that the authorities should avoid incurring any monitoring cost by simply abandoning the idea of creating a recycling market through policy interventions. In particular, the authorities must keep the subsidy at a lower level than the one that requires the monitoring ($s^* \leq \hat{s}$: the optimal levels of subsidy should be in the shaded area of Figure 4). Since the optimal quantity of recycling is zero along with no monitoring at all, only a tax on the product is actually implemented; that is, the optimal policy set becomes the advanced disposal fee (or ADF) in this case.

4 Discussion

In the previous section, we find the second best policy combination for a low-valued recyclable product and show that there are the two distinct types of optimal policy sets depending on the level of environmental damage d: the D-R policy with monitoring that gives birth to the waste market, and the ADF without monitoring that does not create the recycling market. In this section, we further elaborating on this result and argue that, due to the structure of the optimal policy set, these policies are by nature difficult to implement appropriately. One potential consequence is that it becomes quite plausible that the government mistakenly creates the waste market when it should not. As a likely scenario we focus on the case where the social cost of the legally disposed household wastes, i.e., d, gradually increases for certain reasons, such as the increasing shortage of waste disposal sites and the gradual rise in the society's environmental awareness, and discuss the

[18]When the monitoring cost for the firm is infinitesimally small, i.e., Γ'_f is almost zero everywhere, $A(s^M)$ is also close to zero according to the composition of $A(s^M)$. In such a case, therefore, our second-best D-R policy approximates the first-best D-R policy $t = s = d$.

difficulties that are involved in implementing the optimal policies we have proposed above.

If the monitoring cost on the firm is zero (hence, there is no monitoring issue concerning the firm), it is true that the waste market should not emerge if and only if it does not emerge with the first-best tax-subsidy policy, $t = s = d$.[19] Thus, in this ideal case, just by implementing this simple standard two-part policy instrument, the waste market emerges when d reaches the level at which it should emerge. The authorities do not need to worry about exactly when the waste market should be created.

In contrast, when there is a monitoring issue concerning the firm, the structure of the second-best policy implies the following:

Corollary 1. *The residual waste market should not emerge if but not only if it does not emerge with the first-best tax-subsidy policy.*

When $d \leq \bar{d}$, the waste market should not emerge in the second-best situation. However, there is a gap between the marginal social cost of the legally disposed household wastes and the second-best subsidy level in the region where d exceeds $\hat{s}$ (see Figure 4). This is because if the subsidy is provided simply at the level of $s = d$, the authorities must monitor the firm when d exceeds $\hat{s}$ before d reaches $\bar{s}$ (see Lemma 2). Otherwise, the subsidy of the amount $s = d$ induces the firm to participate in the waste market and simply dispose of all the obtained waste illegally. In such a case, the first-best tax-subsidy provision creates the waste market when it should not emerge. In other words, since $\bar{s} \geq \hat{s}$ or $B'(0) < 0$ (see also the paragraph just below Lemma 2), an undesirable waste market emerges. Thus, as regards the desirability of the recycling market, we need some other criteria than this simple two-part instrument.

Remark 1 Even when there is a monitoring issue concerning the firm's behavior, as long as the recycled material has positive net economic value, at least, for certain initial units, i.e., $B'(0) > 0$, Ino (2011) finds it true that zero recycling activity is socially desirable if

[19]Then, illegal disposal by the firm can be prevented, and no monitoring activity for the household is a part of this first-best policy set, i.e., $\tau_h = 0$.

and only if recycling activities does not arise with the subsidy of $s = d$.[20] Hence, there is no concern of creating a socially inefficient waste market by over-encouraging the recycling activities through this simple subsidy scheme. Thus, the above problem is a challenge inherent only for low-valued recyclable products.

As the magnitude of d increases, the policy combination should shift from the advanced disposal fee to the D-R policy, according to the second-best policy set identified in the previous section. In order to implement such a policy shift appropriately, the authorities need to know (i) the level of subsidy $s^M = d - A(s^M)$ that should be included in the D-R policy and (ii) the threshold level in the marginal social cost of the legal household disposal, $\bar{d} \geq \bar{s} + A(\bar{s})$, at which level the change in policy structures need to be undertaken. However, these two critical values contain the information that can be obtained only in the recycling market.[21] In a sense, the authorities face a dilemma in implementing the second-best policy: they need to give birth to the recycling market in order to obtain the information that is necessary to know whether they should create the recycling market and how they should do so. Indeed, the following particular feature of the second-best subsidy scheme is the key property that makes this dilemma quite troublesome.

Corollary 2. *When $B'(0) < 0$, the second-best subsidy level s^* is discontinuous at $d = \bar{d}$.*

If $B'(0) = 0$, the dilemma may not be so serious as the case where $B'(0) < 0$. In this case, s^M approximates the threshold subsidy level which leads to the creation of the desirable recycling market, i.e., $\bar{s} = d - A(\bar{s})$, when d barely exceeds $\bar{d}$, and thus, the second-best subsidy level is continuous at $d = \bar{d}$, as is seen in the right panel of Figure 4. This implies that the waste market first emerges in the minimal scale possible in implementing the second-best policy. In this case, therefore, it is possible for the authorities to induce a small recycling market experimentally and gather the necessary information

[20] See Proposition 2 in Ino (2011). This is because the smallest subsidy level that induces the proper recycling $\bar{s}$ is strictly smaller than the largest subsidy level that does not require the monitoring $\hat{s}$ when $B'(0) > 0$. Thus, before d reaches $\hat{s}$, the subsidy of $s = d$ creates a proper recycling market without monitoring.

[21] As we will see below, even then, the exact level of $\bar{d}$ is not yet known if $B'(0) < 0$.

to find the value of $A(\bar{s})$ without a serious welfare loss.[22] Moreover, the threshold for the policy shift is exactly at $\bar{d} = \bar{s} + A(\bar{s})$ (Proposition 4). Hence, when $A(\bar{s})$ is estimated, $\bar{d}$ can be identified by the same market information.

However, when the recycled material is sufficiently "low-valued" in the sense that $B'(0) < 0$, the second-best subsidy level discontinuously jumps up at $d = \bar{d}$ as is seen in the left panel of Figure 4. This implies that the waste market should suddenly reach a substantial scale as soon as it is launched. Thus, an experimental policy shift could carry a high risk of substantial welfare loss. Furthermore, the threshold social cost for the policy switch is $\bar{d} > \bar{s} + A(\bar{s})$ when $B'(0) < 0$ (see Proposition 4). The market information that is necessary to find $\bar{d}$ is no longer coincides with the information that is necessary to find s^M. This structure of the optimal policy itself renders the authorities quite clueless over whether the creation of the waste market is desirable or not. We might as well call this "the curse of low-valued recycling".[23]

Remark 2 Let us interpret this result more intuitively. When $B'(0) < 0$, proper recycling activities by the firm never start without the authorities' monitoring on the firm's illegal disposal. The fact that $B'(0) < 0$ means that the reprocessing is always unprofitable *per se*. Thus, in order to prevent transacted residual wastes from being disposed illegally by the firm, the monitoring level must be raised sufficiently high to discourage the firm from obtaining the wastes in the first place. On the other hand, when $r^* = 0$, the authorities do not need to monitor the firm and incur the associated monitoring cost. Hence, once the recycling market is created, the authorities are suddenly burdened with substantial monitoring costs. In order to claim that recycling should be encouraged in the presence of the monitoring costs, the social benefits of recycling must be large enough to satisfy:

[22] Since $\bar{s} = P_r(0)$ is the market price when the recycling is minimal, the required market information $A(\bar{s})$ can be obtained once the smallest recycling market emerges.

[23] When $B'(0) > 0$, this "curse" is perfectly avoided. Since we have $\bar{s} < \hat{s}$ in this case, the proper recycling market has already emerged before the monitoring issue arises at $d = \hat{s}$. Thus, the policy-maker does not need to consider the monitoring problem alongside with the desirability of recycling market. Furthermore, in conducting the monitoring, the critical information that is related to the policy shift is $\hat{s}$, which can be obtained by a simple market-based criteria: When $B'(0) > 0$, the subsidy level s exceeds the price for the recycling market if and only if $s > \hat{s}$. See Ino (2011) in detail.

$\bar{d} > \bar{s} + A(\bar{s})$.

5 Concluding Remarks

In this paper, we have identified the second-best deposit-refund policy when there is a possibility that a firm dumps acquired recyclable wastes illegally, especially focusing on the case of low-valued recyclable wastes whose reprocessing itself does not pay financially. As a main result, we found that the structure of the second-best policy is qualitatively different from the one obtained for a case where the recycled material commands a positive net value as is investigated in Ino (2011).

When the social costs associated with its disposal are sufficiently significant, even a fairly low-valued recycling activity can potentially improve the social welfare, which calls for the creation of a residual waste market through some policy measures. However, the mere creation of the market via the subsidy scheme can lead to an enormous amount of illicit disposal by the firm which is supposed to be engaged in reprocessing, due to the negative financial incentive of such an activity. As a result, substantial monitoring efforts by the authorities may become necessary to deter illegal dumping by the firm as the illicitly-dumped wastes are typically more socially-costly than the municipal processing of legally-disposed wastes by the household. Thus, the desirability of the existence of the recycling market critically hinges on the relative magnitudes of the monitoring cost expended by the authorities and the social cost of legally-disposed wastes by the household.

One major issue in implementing the second-best deposit-refund policy for a low-valued recyclable product is that the information only available in the waste market is critical in finding the level of the refund, i.e., the subsidy provided in that market. However, the creation of the proper recycling market could require substantial monitoring activities as was stressed in the last part of the previous section. This could stifle an effort to extract such information from the market experimentally. Even when such a subsidy level is identified, the authorities do not know the exact level of the marginal social cost of the household above which the subsidy should be provided. These issues do not occur if the recycled material has a positive net value, which justifies a simple use of a modified deposit-refund scheme as is elaborated in Ino (2011). This informational difficulty can pose a great challenge in encouraging the recycling of low-value recyclables through policy actions even though the recycling itself is potentially desirable from the society's viewpoint.

As the empirical work by Kinnaman et al. (2014) suggests, the over-encouragement of recycling activities are likely to be prevalent phenomena in many developed nations. Such a finding can be related to the information difficulty in implementing the second-best policy set, which we have discussed in this paper. In the case of low-valued recyclables, the government can be quite clueless about the threshold social cost of household waste disposal and also about the optimal subsidy level, both of which are crucial in implementing the second best policy set. Policies chosen by such an ill-informed government can lead to substantial illegal waste disposal by firms or to an exorbitant monitoring cost if the government wishes to contain them ex post without creating proper recycling markets. This informational issue is unique to low-valued recyclables and can be a big obstacle in achieving the welfare-maximizing outcome.

Appendix

Proof of Lemma 1

Proof. This proof is applicable to both case (i) and case (ii) above. Let the given policy set $(t, s; \tau_f, \tau_h) = (t', s'; \tau'_f, \tau'_h)$ and suppose that $z^*_f(t', s'; \tau'_f, \tau'_h) = z'_f > 0$. We now alter the levels of policy instruments. We depict the starting states in Figure 1. In these states, the following must be satisfied: $P_r(\hat{r}(\tau'_f); \tau'_h) < s' - \tau'_f$ and $r^c = \hat{r}(\tau'_f)$. Reset $s = s' - \{(s' - \tau'_f) - P_r(\hat{r}(\tau'_f); \tau'_h)\}$ and, therefore, $s - \tau'_f = P_r(\hat{r}(\tau'_f); \tau'_h)$. Then, we have $z_f = 0$ and still, $r^c = \hat{r}(\tau'_f)$. Thus, $\Delta r = -z'_f$, where Δ represents the difference between the starting state and the state after the alteration of the policy. Since s does not affect the supply and demand of the product, $-\Delta r = \Delta z$ since $z = x^d - r^s$. In other words, all the firms' illegally disposed residual wastes are converted into the household's legal waste. Thus, $\Delta W = (d_f - d)z'_f + C_r(z'_f + \hat{r}(\tau'_f)) - C_r(\hat{r}(\tau'_f)) > 0$. Note that, because the monitoring level on the firm is unchanged, the monitoring cost is unchanged as well. **Q.E.D.**

Proof of Proposition 1

Proof. Suppose that a set of optimal policies is given by $(t^*, s^*; \tau^*_h) = (t', s'; \tau'_h)$ and $\tau'_h > 0$. Note that, under the optimal policy set, we have $\tau_f = \hat{\tau}_f(s'; \tau'_h)$, and thus $z^*_f = 0$. Then, the equilibrium outcomes under this policy set are $(x, r, z) = (x^*(t'; \tau'_h), r^*(s'; \tau'_h), x^*(t'; \tau'_h) - r^*(s'; \tau'_h))$. Consider the case where the authorities reset the policy $(t, s; \tau_h) = (t' + \tau'_h, s' + \tau'_h; 0)$ under the optimal monitoring rule $\tau_f = \hat{\tau}_f(s' + \tau'_h; 0)$. Then, the equilibrium outcomes in this case are exactly the same as those before the alteration; that is, $x^*(t'; \tau'_h) = x^*(t' + \tau'_h; 0)$ and $r^*(s'; \tau'_h) = r^*(s' + \tau'_h; 0)$ and, thus, the equilibrium amount of the legal disposal by the household, $z^* = x^* - r^*$, is also unchanged. This is because the supply curves for the product and recycling markets both shift up by τ'_h and simultaneously the demand curves for the product and recycling markets both shift up by τ'_h. Furthermore, we have $P^*_r(s' + \tau'_h; 0) - P^*_r(s'; \tau'_h) = \tau'_h$. This implies that $\hat{\tau}_f(s'; \tau'_h) = \hat{\tau}_f(s' + \tau'_h; 0)$ by the definition of the optimal monitoring rule. Since only the monitoring cost for the household is changed and all the other things are constant after the alteration of the policy set, $W^*(t' + \tau'_h, s' +$

$\tau'_h; 0) - W^*(t', s'; \tau'_h) = \Gamma_h(\tau'_h) - \Gamma_h(0)$. Since $\Gamma_h(\tau'_h) > \Gamma_h(0)$ when $\tau'_h > 0$, this policy change leads to the saving in the monitoring cost on the household. This poses a contradiction to the claim that $(t', s'; \tau'_h)$ are optimal. **Q.E.D.**

Proof of Proposition 2

Proof. The first-order conditions of the problem (19) with respect to t is

$$\frac{\partial W^*(t,s)}{\partial t} = U' \frac{\partial x^*}{\partial t} - Cx' \frac{\partial x^*}{\partial t} - d \frac{\partial x^*}{\partial t} = 0. \tag{20}$$

Plugging the first-order conditions, (3) and (10), obtained for the product market into (20) and solving the equation with respect to t, we get $t = d$. **Q.E.D.**

Proof of Proposition 3

Proof. In the first case, we have $r^*(s; 0) > 0$, or equivalently, $s > C'_r(0) - B'(0)$, under the optimal policy set. When $s > C'_r(0) - B'(0) \geq C'_r(0)$, $\hat{\tau}_f(s) = s - P^*_r(s; 0) > 0$ holds by Lemma 2. Therefore, in the first case, the first-order condition of the problem (19) with respect to s becomes:

$$\frac{\partial W^*(t,s)}{\partial s} = B' \frac{\partial r^*}{\partial s} - C'_r \frac{\partial r^*}{\partial s} + d \frac{\partial r^*}{\partial s} - \Gamma'_f \left[1 - \frac{\partial P^*_r}{\partial s} \right] = 0. \tag{21}$$

Plug the first-order conditions for the residual waste market, (4) and (17), into (21) and solve the equation with respect to s, making use of the following comparative statics results derived from (4) and (17):

$$\frac{\partial P^*_r}{\partial s} = \frac{C_r''(r^*)}{C_r''(r^*) - B''(r^*)} > 0, \tag{22}$$

$$\frac{\partial r^*}{\partial s} = \frac{1}{C_r''(r^*) - B''(r^*)} > 0. \tag{23}$$

and, we eventually obtain $s^* = s^M$ in the proposition. In the second case, we have $r^*(s; 0) = 0$, or equivalently, $s \leq C'_r(0) - B'(0)$, under the optimal policy set. When $s \leq C'_r(0) - B'(0)$, $W^*(d, s)$ is decreasing in s if $s > C'_r(0)$ since $r^*(s; 0) = 0$ and $\hat{\tau}_f(s) = s - C_r(0) > 0$; $W^*(d, s)$ is constant in s if $s \leq C'_r(0)$ since $r^*(s; 0) = 0$ and $\hat{\tau}_f(s) = 0$. Therefore, in the second case, the optimal subsidy level is any s such that satisfies $s \leq C'_r(0)$. **Q.E.D.**

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